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Pre-sweeping Algorithm for Solving the Chan-Vese Model in Image Segmentation*

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Abstract: This paper is concerned with a novel and efficient algorithm, the pre-sweeping algorithm, for solving the Chan-Vese model in image segmentation. This algorithm avoids solving the Euler-Lagrange equation corresponding to the energy function. It improves the computational speed dramatically when applied to the Chan-Vese image segmentation model and keeps all the advantages of the level set method such as automatically handling the topological variety, preserving the sharp-angle of the curve, etc. Our algorithm is also efficient for some image that the Song's method does not work on when using the Jacobi iteration. Moreover, our algorithm is easy to extend to arbitrary finite dimensional image segmentation.

Keywords: image segmentation; Chan-Vese model; pre-sweeping algorithm; level set

Classification: AMS(2000) 90C47; 68U10 **CLC number:** TP301.6 **Document code:** A

1 Introduction

Image segmentation is a bridge between image processing and image analysis. The aim is to partition a given image into its constituent parts with same or similar properties, such as intensity, colors, and textures etc. It has been widely used in many fields, for example, the edge detection in computer vision, medical image processing, character recognition, and classification of remote sensing image[2-5].

Variational method is a powerful tool in image processing and other fields such as shape optimization, flow control, material science[6-12]. The basic idea of the variational method is to minimize an energy/cost function depending on the problem to be dealt with. The Kass-Witkin-Terzopoulos model[13] may be the first effort in this direction. Since the energy function in the Kass-Witkin-Terzopoulos model is nonconvex, no uniqueness result is available and the result is much depending on the original contour. Mumford and Shah presented a model[14] with good theoretical properties. The model has difficulty to be applied since it requires to minimize the energy function with two variables in two different natures. Moreover, these models identify boundary with the edge detector. So for the edges which is not defined with gradient, these methods are poor efficient.

To improve the efficiency of the Mumford-Shah model, Chan and Vese[1] proposed a new method introducing level set[9,15,16] into the Mumford-Shah function[14] for image segmentation.

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Experience has shown the efficiency of C-V model for image segmentation.

Many new and efficient methods have been proposed to solve the Chan-Vese model in recent years[17-20]. Among these methods, the fast algorithm proposed by Song and Chan[18] is a novel and efficient one. And it can be applied to broader range of optimization problems without solving PDE. We propose an algorithm which is similar but different from the Song's algorithm.

All the examples considered in this paper are 2-D gray images. It is not meant the method is limited to 2-D images. It is convenient to extend our algorithm to arbitrary finite dimension.

This paper is organized as follows. In the next section, we briefly review the C-V model. In section 3, we introduce a new algorithm, pre-sweeping algorithm, for C-V model. In section 4, we give several examples of our algorithm applied to image segmentation. And we compare it with the algorithm of active contour method (ACM) and the fast algorithm proposed by Song and Chan[18]. Section 5 is the conclusion and some remarks.

2 Chan-Vese model for image segmentation

The C-V model is a variational model for 2-phase image segmentation. From now on, let $\Omega \subset R^2$ be a bounded open set. Curve $C(s) = (x(s), y(s)) : R \to R^2$ divides Ω into two regions: D_1 (inside of $C(s)$) and D_2 (outside of $C(s)$). D_1 and D_2 are open sets. $u_0 : R^2 \to R$ is an original image. A piecewise constant function $u(x)$ is a segmentation result of u_0. $u(x) = c_1(x \in D_1)$ and $u(x) = c_2(x \in D_2)$. Then the energy function is

$$\begin{aligned} F_{CV}(C, c_1, c_2) &= \alpha \cdot \big(\text{length}(C)\big)^p + \beta \cdot \text{area}(D_1) \\ &\quad + \lambda_1 \int_{D_1} |u_0 - c_1|^2 \mathrm{d}x + \lambda_2 \int_{D_2} |u_0 - c_2|^2 \mathrm{d}x, \end{aligned} \tag{1}$$

where α, β, λ_1 and λ_2 are weight parameters. The parameter can be given in advanced according to the image to be processed. In the C-V model, c_1 and c_2 are mean values of $u_0(x)$ on the region of D_1 and D_2, respectively

$$c_1 = \frac{\int_{D_1} u(x)\mathrm{d}x}{\text{meas}(D_1)}, \quad c_2 = \frac{\int_{D_2} u(x)\mathrm{d}x}{\text{meas}(D_2)}. \tag{2}$$

Introducing the Heaviside function $H(z) = 0\,(z < 0)$, $H(z) = 1\,(z \geq 0)$ and a Lipschitz continuous level set function $\phi(x,t) : R^2 \times [0, \infty] \to R$

$$\phi(x,0) \begin{cases} > 0, & x \in D_1, \\ = 0, & x \in C, \\ < 0, & x \in D_2, \end{cases} \tag{3}$$

into the energy function. We use the zero level set of the level set function $\phi(x) : R^2 \to R$ to represent the curve $C(x)$. So the energy function can be written as[1,19,20]

$$\begin{aligned} F(\phi, c_1, c_2) &= \alpha\Big(\int_\Omega |\,H(\phi)\,| \mathrm{d}x \Big)^p + \beta \int_\Omega H(\phi)\mathrm{d}x \\ &\quad + \lambda_1 \int_\Omega |u_0 - c_1|^2 H(\phi)\mathrm{d}x + \lambda_2 \int_\Omega |u_0 - c_2|^2 \big(1 - H(\phi)\big)\mathrm{d}x. \end{aligned} \tag{4}$$

The classical way to solve the problem

$$\min_{\phi(x)} F\big(\phi, c_1(\phi), c_2(\phi)\big)$$

needs to solve the corresponding Euler-Lagrange equation, usually a nonlinear parabolic PDE. Due to the Courant-Friendrichs-Lewy (CFL) condition, the numerical algorithm is often not efficient. Song and Chan[18] proposed a fast algorithm without solving PDE. The algorithm is efficient and converges very fast. Motivated by this algorithm, we propose an algorithm that is similar but different from the Song and Chan's algorithm.

3 Pre-sweeping algorithm for Chan-Vese model

We suppose that the level set function $\phi(x) = 0$ divides Ω into two different regions: D_1 and D_2. Let k_1 and k_2 be the number of pixels in D_1 and D_2, respectively, and c_1 and c_2 be mean value of $u(x)$ in the D_1 and D_2. Let

$$\phi(x) = 1(x \in D_1), \quad \phi(x) = -1(x \in D_2), \quad D_0 = \{x \mid \phi(x) = 0;\ x \in \Omega\}.$$

We use k_0 and c_0 to present the pixels number belonging to D_0 and the mean value of D_0, respectively.

Let $\widetilde{H}(z) = 1\,(z > 0)$ and $\widetilde{H}(z) = 0\,(z \le 0)$. We rewrite the energy function the Chan-Vese model as follows

$$\begin{aligned} F\big(\phi(x)\big) \;=\; & \lambda_1 \int_\Omega |\, u_0 - c_1\,|^2 \widetilde{H}(\phi)\mathrm{d}x + \lambda_2 \int_\Omega |\, u_0 - c_2\,|^2 \big(\widetilde{H}(-\phi)\big)\mathrm{d}x \\ & + \lambda_0 \int_{D_0} |\, u_0(x) - c_0\,|^2 \mathrm{d}x + \mu \int_\Omega |\,\nabla \widetilde{H}(\phi)\,| \mathrm{d}x. \end{aligned} \tag{5}$$

Where $\int_{D_0} f\mathrm{d}x$ represents the integral on D_0. D_0 may be a curve or a surface. Then the image segmentation problem is to minimize above function in piecewise constant function (PCF)

$$\min_{\phi \in PCF} F(\phi). \tag{6}$$

Set the parameters $\lambda_0 = \lambda_1 = \lambda_2 = 1$ and $\mu = 0$. Let $\triangle F_1$ and $\triangle F_2$ be the varieties of the energy function (5) when we change a point $x_0 \in D_0$ to D_1 and to D_2, respectively. If we change $\phi(x_0) = 0$ to $\phi(x_0) = -1$, the new energy is

$$\tilde{F_2}(x_0) = \sum_{x \in D_1} |u_0(x) - c_1|^2 + \sum_{x \in (D_2 \bigcup x_0)} |u_0(x) - \tilde{c}_2|^2 + \sum_{x \in (D_0 \backslash x_0)} |u_0(x) - \tilde{c}_0|^2, \tag{7}$$

where

$$\tilde{c}_2 = \frac{k_2 c_2 + u_0(x_0)}{k_2 + 1}, \quad \tilde{c}_0 = \frac{k_0 c_0 - u_0(x_0)}{k_0 - 1}.$$

So the difference between the new energy and the old energy is

$$\triangle F_2(x_0) = \tilde{F_2}(x_0) - F(x) = \big(u_0(x_0) - c_2\big)^2 \frac{k_2}{k_2 + 1} - \big(u_0(x_0) - c_0\big)^2 \frac{k_0}{k_0 - 1}. \tag{8}$$

Similarly, it is easy to calculate

$$\triangle F_1(x_0) = \big(u_0(x_0) - c_1\big)^2 \frac{k_1}{k_1+1} - \big(u_0(x_0) - c_0\big)^2 \frac{k_0}{k_0-1}. \tag{9}$$

Our pre-sweeping algorithm for solving the problem (6) is:

Initialize For a given image u_0 and an initial curve $\phi(x)$, we calculate the c_1 and c_2. Without loss of generality, we assume $c_1 \geq c_2$. Set $\phi(x) = 1$ if $x \in D_1$, $\phi(x) = -1$ if $x \in D_2$, and $D_0 = x : \phi(x) = 0$.

Step 1 Pre-sweeping. For every $x \in \Omega$, if $\phi(x) = 1$ and $u(x) \leq c_2$, then change $\phi(x) = -1$; else if $c_2 \leq u(x) \leq c_1$, change $\phi(x) = 0$. If $\phi(x) = -1$ and $u(x) \geq c_1$, then change $\phi(x) = 1$; else if $c_2 \leq u(x) \leq c_1$, change $\phi(x) = 0$.

Step 2 Sweeping. For every point $x \in D_0$, compute $\triangle F_1(x)$ and $\triangle F_2(x)$ with (8) and (9). If $\triangle F_1 \geq 0$ and $\triangle F_2 \geq 0$, then stop; otherwise change $\phi(x)$ to 1 or -1 according to the less $\triangle F_i(x)$. Then go to the beginning of Step 2.

Theorem 3.1 If $u_0 : \Omega \to R$ is a 2-phase image and satisfies

$$u_0(x) = \begin{cases} a, & x \in A, \\ b, & x \in B, \end{cases} \tag{10}$$

where a and b are constants $(a > b)$. Then for any initial curve $\phi(x) = 0$ dividing Ω into two different regions, the pre-sweeping algorithm converges to the exact result after pre-sweeping for the model (5) with $\lambda_0 = \lambda_1 = \lambda_2 = 1$ and $\mu = 0$.

Proof Suppose that the curve $\phi(x) = 0$ divides Ω into D_1 and D_2. c_1 and c_2 are the mean values of D_1 and D_2, respectively. Without lose of generality, we assume $c_1 \geq c_2$, then we have $b \leq c_2 \leq c_1 \leq a$. Set $\phi(x) = 1$ if $x \in D_1$ and $\phi(x) = -1$ if $x \in D_2$.

If $x \in D_1$ and $u_0(x) \leq c_2$, so we have $u_0(x) = b$. According to the algorithm, we change $\phi(x) = 1$ to $\phi(x) = -1$. If $x \in D_2$ and $u_0(x) \geq c_1$, so we have $u_0(x) = a$. Using the pre-sweeping algorithm, we change $\phi(x) = -1$ to $\phi(x) = 1$.

So after the pre-sweeping, $\phi(x) = -1$ for $x : u(x) = b$ and $\phi(x) = 1$ for $x : u_0(x) = a$.

After pre-sweeping, we sweep on the D_0 by using the following theorem:

Theorem 3.2 Suppose $D_0 \neq \varnothing$. Let

$$A = \big\{x \mid u(x_0) \leq u(x) \leq 2c_0 - u(x_0)\big\}, \quad B = \big\{x \mid u(x_0) \geq u(x) \geq 2c_0 - u(x_0)\big\}.$$

1) If x_0 is a solution of $\min\limits_{x \in D_0} \{u(x)\}$, and the energy function (5) is non-decreasing when changing $\phi(x_0) = 0$ to $\phi(x_0) = -1$, then the energy function is non-decreasing for every $x \in A$ changing to $\phi(x) = -1$.

2) If x_0 is a solution of $\max\limits_{x \in D_0} \{u(x)\}$, and the energy function (5) is non-decreasing when changing $\phi(x_0) = 0$ to $\phi(x_0) = 1$, then the energy function is non-decreasing for every $x \in B$ changing to $\phi(x) = 1$.

Proof We only need to prove the 1). 2) can be proved in the same way.

1) The energy function (8) is non-decreasing when changing $\phi(x_0) = 0$ to $\phi(x_0) = -1$, so $\triangle F_2(x_0) \geq 0$. For every $x \in A$, we have

$$c_2 \leq u(x_0) \leq c_0, \quad |c_0 - u(x_0)| \geq |u(x) - c_0|, \quad |c_2 - u(x_0)| \leq |u(x) - c_2|.$$

So we have

$$\begin{aligned}\triangle F(x) &= \left(u(x)-c_2\right)^2\frac{k_2}{k_2+1}-\left(u(x)-c_0\right)^2\frac{k_0}{k_0-1}\\ &\geq \left(u(x_0)-c_2\right)^2\frac{k_2}{k_2+1}-\left(u(x_0)-c_0\right)^2\frac{k_0}{k_0-1}\geq 0. \end{aligned} \tag{11}$$

In the above theorem, we have not considered the $x : u(x) > 2c_0 - u(x_0)$ when changing $\phi(x_0) = 0$ to $\phi(x_0) = -1$ and the $x : u(x) < 2c_0 - u(x_0)$ when changing $\phi(x_0) = 0$ to $\phi(x_0) = 1$. It is obvious that we want to move the x with lager value to the domain D_1 and the small value to D_2. So the method is reasonable.

4 Numerical experimental result

In this section, we present several examples to show the application of our pre-sweeping algorithm for image segmentation. For simplicity, we set $\lambda_0 = \lambda_1 = \lambda_2 = 1$ and $\mu = 0$ in all the examples. The pre-sweeping algorithm is very easy to program. All of the examples in the paper are executed in a personal computer with CPU Pentium IV 3.40 G Hz and 512 M memory. We program with Matlab.

In Figure 1 and Figure 2, we applied three different algorithms, our pre-sweeping algorithm, the Song and Chan's fast algorithm[18], and the active contour method without edge[1], to an image named cameramen with the same original level sets all being given as a circle. This example shows that our algorithm and Song's algorithm are almost convergence to the same result. On the other hand, we will show the proposed algorithm is more efficient than the fast algorithm proposed by Song's later on.

In Figure 2, we use the active contour without edge method to the same image. The result shows that the active contour without edge method converges very slowly. We show the segmentation results iteration in 1000, 6000 and 10000 iterations. And it requires almost ten thousands times iteration if one wants to get the similar result as the former two methods got.

(a) (b) (c) (d)

Figure 1: (a) The cameramen image with the initial level set; (b) and (c) are segmentation results of the cameramen image using the pre-sweeping algorithm and the Song's algorithm, respectively; (d) The result of level set using pre-sweeping algorithm.

(a) (b) (c) (d)

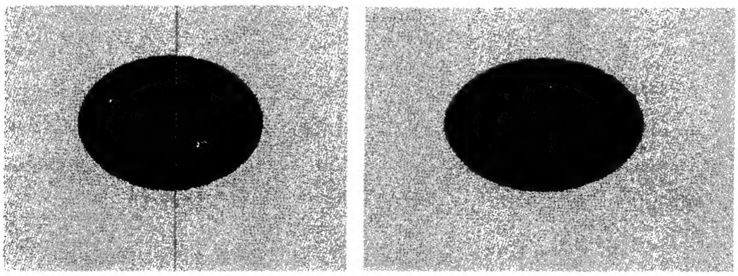
Figure 2: (a) The image with the initial level set; (b), (c) and (d) are the level sets after 1000, 6000, and 10000 iterations, respectively.

Table 1 compares our pre-sweeping algorithm with the Song's algorithm in terms of CPU times. We apply the two algorithms to the same image with different sizes. The bigger size the image is, the more advantage our algorithm shows.

Table 1: Comparison of CPU times

image size	128× 128	256×256	512×512	1024×1024
pre-sweeping algorithm	0.1879s	0.2399s	0.5278s	1.6844s
Song and Chan's algorithm	0.2506s	2.5108s	30.8046s	237.3616s

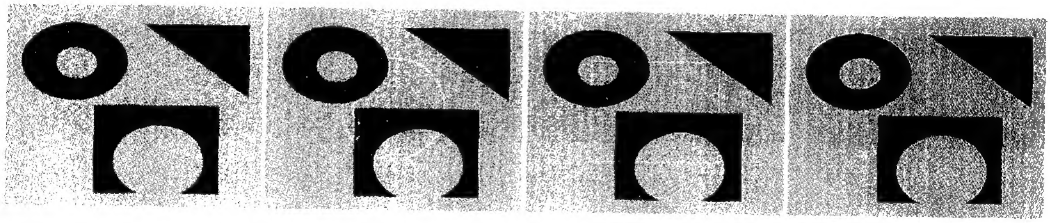
In Figure 3, we give an image which the Song's algorithm with the Jacobi iteration does not work. If the initial level set function is $\phi = 1$ on the left side and $\phi = -1$ on the right side, using the Song's method with Jacobi iteration, when a point x in left can be changed to $\phi(x) = -1$ or $\phi(x) = 1$, the point on the right corresponding would also be changed to $\phi(x) = 1$ or $\phi(x) = -1$ for symmetry. So the Song's algorithm failed for this kind of images. While our pre-sweeping algorithm can converge to the accurate result whatever using the Jacobi iteration or the Gauss iteration.



(a) (b)

Figure 3: (a) A type of symmetrical images that the Song's algorithm does not work when using the Jacobi iteration; (b) The segmentation result of our pre-sweeping algorithm.

In figure 4, we use our algorithm to segment an image with three different initial conditions. All of them converge to the accurate result. This shows our algorithm is quite robust. And it can be seen that the interior contour of the objects can be detected automatically. Even for the non-convex contour, it can also be detected correctly.



(a) (b) (c) (d)

Figure 4: To segment an image using our pre-sweeping algorithm with three different original conditions (a), (b) and (c); all of them get the same result which is shown in (d).

In Figure 5, we show an image with three different objects. It requires two level sets to divide the image correctly when using the Song's algorithm. Our algorithm only needs one level set to get the correct result.

(a) (b)

Figure 5: We use one level set to divide an image into three different parts. (a) is the original image with initial level set; (b) is the result. In this case, the level set $\phi = 0$ represents the third part.

The results of our algorithm used to the noised image are shown in the Figure 6. We use the pre-sweeping algorithm to the image with 5 percent salt and pepper noise. The two results are shown together with the same initial level set. From the figures we can see that our algorithm can get satisfied segmentation result even for the image with noise.

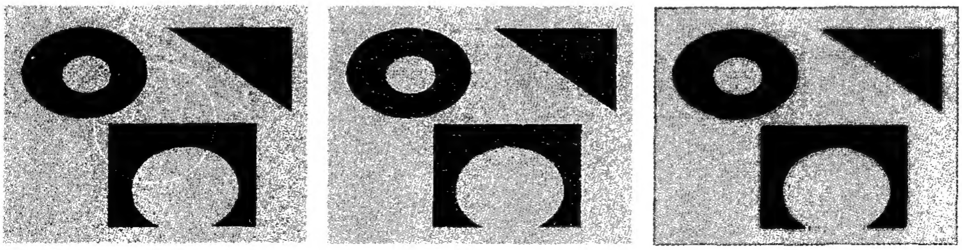
5 Conclusion and some remarks

We proposed a new algorithm, the pre-sweeping algorithm, in this paper. The new algorithm is very efficient and robust. It has many advantages compared with other algorithms used to solve the Chan-Vese model in image segmentation. The algorithm preserves the advantage of

the level set method. Our algorithm doesn't require to solve any PDE. So using our algorithm, it doesn't need to consider the CFL condition. This makes our algorithm to converge faster than the PDE based methods. For some 2-phase images, our algorithm only needs pre-sweeping to get satisfied result. And for some images which have three different parts, the pre-sweeping algorithm only needs one level set to get correct result.

All the examples in our paper are 2-D images, but the algorithm can be extended to high dimension image segmentation easily. Moreover, the algorithm can be used to a lot of optimization problems when the energy function could be easily calculated.

Since we need not calculate the total energy, the length term in (5) is ignored in our algorithm. So the method is sensitively for noised image. But we can get satisfactory result using the pre-sweeping to the denoised image.



(a) (b) (c)

Figure 6: (a) The original image added with 5% salt-and-pepper noise with initial level set; (b) The segmentation result of pre-sweeping algorithm to the noisy image directly; (c) The segmentation result of the image after average filtering.

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预扫描算法求解图像分割的Chan-Vese模型

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摘 要: 本文提出了一种新的有效的算法来求解图像分割中的Chan-Vese模型。新算法避免了求解PDE的过程，极大地提高了图像分割的运算速度。这种算法保持了C-V模型和水平集方法的优点，能够自动处理图像分割过程中边缘的拓扑变形，保持边缘的尖角以及对于非凸边缘的有效的检测等等。这种算法思路简单，很容易推广到任意有限维的图像分割问题的求解中。

关键词: 预扫描算法；Chan-Vese模型；图像分割；水平集